

Masses and magnetic moments of pentaquarks

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Abstract

We discuss the spectroscopy of $q^4\bar{q}$ pentaquarks. The quantum numbers of the ground state depend on the interplay between spin-flavor and orbital contributions to the energy. The magnetic moments of the lowest pentaquark state with negative and positive parity are found to be $0.382 \mu_N$ and $0.089 \mu_N$, respectively.

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1 Introduction

The discovery of the $\Theta^+(1540)$ resonance with positive strangeness $S = +1$ by the LEPS Collaboration [1] and the subsequent confirmation by various other experimental collaborations [2, 3, 4, 5, 6] has sparked an enormous amount of experimental and theoretical studies of exotic baryons [7]. Since there are no three-quark configurations with these properties, it is an exotic baryon resonance, whose simplest configuration is that of a $q^4\bar{q}$ pentaquark consisting of four quarks and an antiquark. Its other quantum numbers are still mostly unknown, although the absence of a signal for a Θ^{++} pentaquark state in the pK^+ invariant mass spectrum is an indication that the observed Θ^+ is most likely to be an isosinglet $I = 0$ [3, 4, 6]. The width of the Θ^+ is so small that only an upper limit could be established. A re-analysis of elastic K^+N scattering data has put an even more stringent upper bound on the width of only a few MeV [8] which suggests that it is exotic dynamically as well as in its quantum numbers. More recently, evidence has been found for the existence of another exotic baryon $\Xi^{--}(1862)$ with strangeness $S = -2$ by the NA49 Collaboration [9]. The Θ^+ and Ξ^{--} resonances are interpreted as pentaquarks belonging to a flavor antidecuplet with isospin $I = 0$ and $I = 3/2$, respectively.

The spin and parity of the Θ^+ have not yet been determined experimentally. Theoretically, there seems to be agreement between different approaches on the value of the angular momentum, i.e. $J = 1/2$, but not so on the parity. The parity of the lowest pentaquark state is predicted to be positive by many studies, such as chiral soliton models [10], cluster models [11, 12, 13] and various constituent quark models [14]. However, there is also evidence for a negative parity ground state pentaquark from recent work on QCD sum rules [15], lattice QCD [16] and a quark model calculation [17]. Various suggestions have been made for future experiments to measure the parity [18, 19, 20, 21].

Another unknown quantity is the magnetic moment. Although it may be difficult to determine its value experimentally, it is an essential ingredient in calculations of the photo- and electroproduction cross sections [19, 20, 21]. In the absence of experimental information, one has to rely on model calculations.

The aim of this article is to study the spin, parity and magnetic moment of the ground state pentaquark. First, we construct a complete classification scheme of pentaquark states based on the spin-flavor $SU(6)$ symmetry. Some general characteristics of the spectrum are discussed for a schematic harmonic oscillator quark model. Next, we derive the magnetic moments of the lowest exotic pentaquarks with negative and positive parity. Finally, it is shown that in a string-like algebraic model the ground state has negative parity.

2 Pentaquark states

We consider pentaquarks to be built of five constituent parts which are characterized by both internal and spatial degrees of freedom (see Fig. 1).

2.1 Internal degrees of freedom

The internal degrees of freedom are taken to be the three light flavors u, d, s with spin $s = 1/2$ and three possible colors r, g, b . The corresponding algebraic structure consists of the usual spin-flavor and color algebras

$$\mathcal{G}_i = SU_{\text{sf}}(6) \otimes SU_c(3) . \quad (1)$$

The pentaquark spin-flavor states can be classified according to the irreducible representations of $SU_{\text{sf}}(6)$ and its subgroups

$$\left| \begin{array}{ccccccc} SU_{\text{sf}}(6) & \supset & SU_{\text{f}}(3) & \otimes & SU_{\text{s}}(2) & \supset & SU_{\text{I}}(2) \otimes U_{\text{Y}}(1) \otimes SU_{\text{s}}(2) \\ [f] & , & [g] & , & [h] & , & I \quad , \quad Y \end{array} \right\rangle , \quad (2)$$

where I and Y denote the isospin and hypercharge.

In the construction of the classification scheme we are guided by two conditions: the pentaquark wave function should be a color singlet, and should be antisymmetric under any permutation of the four quarks. The permutation symmetry of the four-quark subsystem is characterized by the S_4 Young tableaux [4], [31], [22], [211] and [1111] or, equivalently, by the irreducible representations of the tetrahedral group \mathcal{T}_d (which is isomorphic to S_4) as A_1, F_2, E, F_1 and A_2 , respectively. For notational purposes we use the latter to label the discrete symmetry of the pentaquark wave functions. The corresponding dimensions are 1, 3, 2, 3 and 1. The allowed spin, flavor and spin-flavor states are obtained by standard group theoretic techniques (see Table 1). The full decomposition of the spin-flavor states into spin and flavor states is given in Table 5 of [22]. The states of a given flavor multiplet can be labeled by isospin I, I_3 and hypercharge Y . The electric charge is given by the Gell-Mann-Nishijima relation

$$Q = I_3 + \frac{Y}{2} = I_3 + \frac{B + S}{2} , \quad (3)$$

where B denotes the baryon number and S the strangeness. It is difficult to distinguish the pentaquark flavor singlets, octets and decuplets from the three-quark flavor multiplets, since they have the same values of the hypercharge Y and isospin projection I_3 . The same observation holds for the majority of the states in the remaining flavor states. However, the antidecuplets, the 27-plets and 35-plets contain in addition exotic states which cannot be obtained from three-quark configurations. These states are more easily identified experimentally due to the uniqueness of their quantum numbers. As an example, the exotic states of the antidecuplet are indicated by a \bullet in Fig. 2: the Θ^+ is the isosinglet $I = I_3 = 0$ with hypercharge $Y = 2$ (strangeness $S = +1$), and the cascades $\Xi_{3/2}^+$ and $\Xi_{3/2}^-$ have hypercharge $Y = -1$ (strangeness $S = -2$) and isospin $I = 3/2$ with projection $I_3 = 3/2$ and $-3/2$, respectively.

2.2 Spatial degrees of freedom

The relevant degrees of freedom for the relative motion of the constituent parts are provided by the Jacobi coordinates which we choose as [23]

$$\begin{aligned}\vec{\rho}_1 &= \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2) , \\ \vec{\rho}_2 &= \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3) , \\ \vec{\rho}_3 &= \frac{1}{\sqrt{12}}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3 - 3\vec{r}_4) , \\ \vec{\rho}_4 &= \frac{1}{\sqrt{20}}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 - 4\vec{r}_5) ,\end{aligned}\tag{4}$$

where \vec{r}_i ($i = 1, \dots, 4$) denote the coordinate of the i -th quark, and \vec{r}_5 that of the antiquark. The last Jacobi coordinate is symmetric under the interchange of the quark coordinates, and hence transforms as A_1 under \mathcal{T}_d ($\sim S_4$), whereas the first three transform as three components of F_2 [23].

2.3 Pentaquark wave functions

The pentaquark wave function is obtained by combining the spin-flavor part with the color and orbital parts in such a way that the total wave function is a color-singlet state, and that the four quarks satisfy the Pauli principle, i.e. are antisymmetric under any permutation of the four quarks. Since the color part of the pentaquark wave function is a $[222]$ singlet and that of the antiquark a $[11]$ anti-triplet, the color wave function of the four-quark configuration is a $[211]$ triplet which has F_1 symmetry under \mathcal{T}_d . The total q^4 wave function is antisymmetric (A_2), hence the orbital-spin-flavor part has to have F_2 symmetry

$$\psi_{A_2} = [\psi_{F_1}^c \times \psi_{F_2}^{\text{osf}}]_{A_2} .\tag{5}$$

Here the square brackets $[\dots]$ denote the tensor coupling under the tetrahedral group \mathcal{T}_d . In Table 2, we present the allowed spin-flavor multiplets with exotic pentaquarks for some lowlying orbital excitations. The exotic spin-flavor states associated with the S -wave state $L_t^p = 0_{A_1}^+$ all belong to the $[f]_t = [42111]_{F_2}$ spin-flavor multiplet. The corresponding orbital-spin-flavor wave function is given by

$$\psi_{F_2}^{\text{osf}} = [\psi_{A_1}^o \times \psi_{F_2}^{\text{sf}}]_{F_2} .\tag{6}$$

A P -wave radial excitation with $L_t^p = 1_{F_2}^-$ gives rise to exotic pentaquark states of the $[51111]_{A_1}$, $[42111]_{F_2}$, $[33111]_E$ and $[32211]_{F_1}$ spin-flavor configurations. They are characterized by the orbital-spin-flavor wave functions

$$\psi_{F_2}^{\text{osf}} = [\psi_{F_2}^o \times \psi_t^{\text{sf}}]_{F_2} ,\tag{7}$$

with $t = A_1, F_2, E$ and F_1 , respectively.

3 Harmonic oscillator quark model

In a harmonic oscillator treatment, the group structure related to the four relative coordinates of Eq. (4) is

$$\mathcal{G}_r = U(12) \supset U(9) \otimes U(3) ,\tag{8}$$

where $U(9)$ describes the relative motion of the four quarks and $U(3)$ that of the antiquark with respect to the four-quark subsystem. We consider a simple schematic model for the spectrum of exotic pentaquarks in which the Hamiltonian is given by

$$H = H_{\text{orb}} + H_{\text{sf}} . \quad (9)$$

Here H_{orb} describes the orbital motion of the pentaquark in terms of harmonic oscillators

$$H_{\text{orb}} = \frac{1}{2}\epsilon_1 \sum_{i=1}^3 (\vec{p}_{\rho_i}^2 + \vec{\rho}_i^2) + \frac{1}{2}\epsilon_2 (\vec{p}_{\rho_4}^2 + \vec{\rho}_4^2) . \quad (10)$$

The first term comes from the three degenerate three-dimensional harmonic oscillators to describe the relative motion of the four quarks, and the second one from the three-dimensional harmonic oscillator for the relative motion of the antiquark with respect to the four-quark system. The energy eigenvalues of H_{orb} are

$$E_{\text{orb}} = \epsilon_1 \left(n_1 + \frac{9}{2} \right) + \epsilon_2 \left(n_2 + \frac{3}{2} \right) . \quad (11)$$

The ground state is an S -wave state with $L^p = 0^+$ and A_1 symmetry for the four quarks. Since the orbital excitations are described by four relative coordinates, there are four excited P -wave states with $L^p = 1^-$, three of which correspond to excitations in the relative coordinates of the four quarks, and the fourth to an excitation in the relative coordinate of the four-quark system and the antiquark. As a consequence of the discrete symmetry of the four quarks, the first three excitations form a degenerate triplet with three-fold F_2 symmetry, and the fourth has A_1 symmetry (see the left-hand side of Figs. 3 and 4). The second term in Eq. (9) represents the spin-flavor dependence of the masses in terms of a generalized Gürsey-Radicati form

$$\begin{aligned} H_{\text{sf}} = & -A C_{2SU_{\text{sf}}(6)} + B C_{2SU_{\text{f}}(3)} + C C_{2SU_{\text{s}}(2)} \\ & + D C_{1U_{\text{Y}}(1)} + E [C_{2SU_{\text{I}}(2)} - \frac{1}{4} C_{1U_{\text{Y}}(1)}^2] , \end{aligned} \quad (12)$$

with eigenvalues

$$\begin{aligned} E_{\text{sf}} = & -\frac{1}{2}A \left[\sum_{i=1}^6 f_i(f_i + 7 - 2i) - \frac{1}{6} \left(\sum_{i=1}^6 f_i \right)^2 \right] \\ & + \frac{1}{2}B \left[\sum_{i=1}^3 g_i(g_i + 4 - 2i) - \frac{1}{3} \left(\sum_{i=1}^3 g_i \right)^2 \right] \\ & + C s(s+1) + D Y + E [I(I+1) - \frac{1}{4} Y^2] . \end{aligned} \quad (13)$$

The contribution from the flavor part is usually expressed in terms of $p = g_1 - g_2$ and $q = g_2 - g_3$ to obtain

$$\frac{1}{3}B(p^2 + q^2 + 3(p+q) + pq) . \quad (14)$$

For the definition of the Casimir operators in Eq. (12), we have followed the same convention as by Helminen and Riska in [14]. The last two terms correspond to the Gell-Mann-Okubo mass formula that describes the splitting within a flavor multiplet [24]. This formula was extended by Gürsey and Radicati

[25] to include the terms proportional to B and C that depend on the spin and the flavor representations, which in turn was generalized further to include the spin-flavor term proportional to A as well [26].

The energy of a given spin-flavor multiplet depends on the oscillator frequencies ϵ_1 and ϵ_2 , and the coefficient A , while the terms proportional to B , C , D and E give the splitting inside the multiplet. The sign of the coefficient A is taken to be positive corresponding to an attractive spin-flavor hyperfine interaction [27], in agreement with the sign used in previous studies of baryons as qqq configurations [26]. The ground state configuration depends on the relative size of ϵ_1 and A . Its parity is opposite to that of the orbital excitation due to the negative intrinsic parity of $q^4\bar{q}$ configurations. For $A < \epsilon_1$, the ground state pentaquark is associated with the orbital state $(n_1, n_2) = (0, 0)$, $L_t^p = 0_{A_1}^+$ and the spin-flavor multiplet $[42111]_{F_2}$, and therefore has negative parity (see Fig. 3). For $A > \epsilon_1$, the parity of the lowest pentaquark state is positive, since the ground state now corresponds to the orbital excitation $(n_1, n_2) = (1, 0)$, $L_t^p = 1_{F_2}^-$ and the spin-flavor state $[51111]_{A_1}$ (see Fig. 4).

The terms proportional to B , C , D and E only contribute to the splitting within a spin-flavor multiplet. They can be estimated from mass differences between baryon resonances as

$$\begin{aligned} 3B + 3C + 3E &= [M(\Delta(1232)) - M(N(938))] , \\ C &= \frac{1}{3} [M(N(1650)) - M(N(1535))] , \\ D &= \frac{1}{4} [4M(N(938)) - M(\Sigma(1193)) - 3M(\Lambda(1116))] , \\ E &= \frac{1}{2} [M(\Sigma(1193)) - M(\Lambda(1116))] . \end{aligned} \quad (15)$$

This gives the numerical values $B = 21.3$ MeV, $C = 38.3$ MeV, $D = -197.3$ MeV and $E = 38.5$ MeV. For the ground state spin-flavor configurations $[51111]_{A_1}$ and $[42111]_{F_2}$ in Figs. 3 and 4, respectively, the lowest exotic pentaquark state has spin $s = 1/2$ and belongs to the antidecuplet [22].

In summary, the parity of the ground state exotic pentaquark depends on the relative contribution of the orbital and spin-flavor parts of the mass operator. We find that if the splitting due to the $SU_{sf}(6)$ spin-flavor term is large compared to that between the orbital states, the ground state pentaquark has positive parity, whereas for a relatively small spin-flavor splitting the parity of the lowest pentaquark state becomes negative.

4 Magnetic moments

In this section, we study the magnetic moments of the lowest exotic pentaquark configuration with positive and negative parity. Which one of these is the ground state, depends on the relative size of the orbital excitations and the spin-flavor splittings.

The magnetic moment is a crucial ingredient in calculations of the photo- and electroproduction cross sections of pentaquarks [19, 20, 21]. A compilation of theoretical values for the chiral soliton model, different correlated quark models, the MIT bag model and for QCD sum rules has been presented in [28]. To the best of our knowledge, the present calculation is the first one for an uncorrelated or constituent quark model.

The magnetic moment of a multi-quark system is given by the sum of the magnetic moments of its constituent parts

$$\vec{\mu} = \vec{\mu}_{\text{spin}} + \vec{\mu}_{\text{orb}} = \sum_i \mu_i (2\vec{s}_i + \vec{\ell}_i) , \quad (16)$$

where $\mu_i = e_i/2m_i$, e_i and m_i represent the magnetic moment, the electric charge and the constituent mass of the i -th (anti)quark. The quark magnetic moments μ_u , μ_d and μ_s are determined from the proton, neutron and Λ magnetic moments to be $\mu_u = 1.852 \mu_N$, $\mu_d = -0.972 \mu_N$ and $\mu_s = -0.613 \mu_N$ [29]. The magnetic moments of the antiquarks satisfy $\mu_{\bar{q}} = -\mu_q$.

4.1 Negative parity pentaquark

The lowest negative parity pentaquark state belongs to the $[f]_t = [42111]_{F_2}$ spin-flavor multiplet, is associated with the orbital state $L_t^p = 0_{A_1}^+$ and has angular momentum and parity $J^p = 1/2^-$ (see Fig. 3). The corresponding wave function is given by

$$\psi_{A_2} = \left[\psi_{F_1}^c \times [\psi_{A_1}^o \times \psi_{F_2}^{sf}]_{F_2} \right]_{A_2}, \quad (17)$$

where the spin-flavor part can be expressed as a product of the antidecuplet flavor wave function ϕ_E and the $s = 1/2$ spin wave function χ_{F_2}

$$\psi_{F_2}^{sf} = [\phi_E \times \chi_{F_2}]_{F_2}. \quad (18)$$

Since the orbital wave function has $L_t^p = 0_{A_1}^+$, the magnetic moment only depends on the spin part. For the Θ^+ , $\Xi_{3/2}^+$ and $\Xi_{3/2}^-$ exotic states we obtain

$$\begin{aligned} \mu_{\Theta^+} &= \frac{1}{3}(2\mu_u + 2\mu_d + \mu_s) = 0.382 \mu_N, \\ \mu_{\Xi_{3/2}^{--}} &= \frac{1}{3}(\mu_u + 2\mu_d + 2\mu_s) = -0.439 \mu_N, \\ \mu_{\Xi_{3/2}^+} &= \frac{1}{3}(2\mu_u + \mu_d + 2\mu_s) = 0.502 \mu_N. \end{aligned} \quad (19)$$

These results are independent of the orbital wave functions, and are valid for any quark model in which the eigenstates have good $SU_{sf}(6)$ spin-flavor symmetry. In the limit of equal quark masses $m_u = m_d = m_s$, the magnetic moments become proportional to the electric charge $\mu_{\Xi_{3/2}^{--}} = -2\mu_{\Xi_{3/2}^+} = -2\mu_{\Theta^+}$.

4.2 Positive parity pentaquark

The lowest positive parity pentaquark has quantum numbers $[f]_t = [51111]_{A_1}$ with orbital excitation $L_t^p = 1_{F_2}^-$ and angular momentum and parity $J^p = 1/2^+$ (see Fig. 4). The wave function is now given by

$$\psi_{A_2} = \left[\psi_{F_1}^c \times [\psi_{F_2}^o \times \psi_{A_1}^{sf}]_{F_2} \right]_{A_2}, \quad (20)$$

where the spin-flavor part can be expressed as a product of the antidecuplet flavor wave function ϕ_E and the $s = 1/2$ spin wave function χ_E

$$\psi_{A_1}^{sf} = [\phi_E \times \chi_E]_{A_1}. \quad (21)$$

In this case, there are contributions to the magnetic moment from both the orbital angular momentum and the spin. Whereas the spin part does not depend on the orbital wave functions, the orbital part obviously does. As a result, the magnetic moments of the positive parity exotic pentaquarks Θ^+ , $\Xi_{3/2}^+$ and $\Xi_{3/2}^-$ are all equal

$$\mu_{\Theta^+} = \mu_{\Xi_{3/2}^{--}} = \mu_{\Xi_{3/2}^+} = \frac{1}{3}(\mu_u + \mu_d + \mu_s) = 0.089 \mu_N. \quad (22)$$

In this calculation we have used harmonic oscillator wave functions with $N = 1$. In the limit of equal quark masses $m_u = m_d = m_s$, the moments vanish due to a cancellation between the spin and orbital contributions.

4.3 Results

The results obtained for the magnetic moments are valid for any quark model in which the eigenstates have good $SU_{\text{sf}}(6)$ spin-flavor symmetry. The magnetic moments for negative parity pentaquarks of Eq. (19) are typically an order of magnitude smaller than the proton magnetic moment, whereas for positive parity they are even smaller due to a cancellation between orbital and spin contributions, see Eq. (22). The magnetic moment of the Θ^+ pentaquark is found to be $0.382 \mu_N$ for negative parity, and $0.089 \mu_N$ for positive parity.

In Table 3 we present a comparison with the theoretical predictions of the magnetic moments of exotic pentaquarks for the chiral soliton model [30], a diquark-diquark-antiquark bound state [19, 20, 28], a diquark-triquark bound state [19, 28] the MIT bag model [20, 28], and for light cone QCD sum rules [31]. Although the different models show some variations in the numerical values, generally speaking, the results for the Θ^+ are relatively close, especially in comparison with the magnetic moment of the proton they are all small. For the Ξ cascade pentaquarks there is a larger spread in the theoretical values which is mostly due to the result obtained for the $\Xi_{3/2}^-$ in the MIT bag model. The magnetic moments for positive parity pentaquarks tend to be smaller than those for negative parity.

5 Stringlike algebraic model

After the schematic calculation of the previous section, we discuss now briefly the mass spectrum of exotic pentaquarks in a stringlike algebraic model. Hadronic spectra are characterized by the occurrence of linear Regge trajectories, i.e. $M^2 \sim \alpha L$, with almost identical slopes for baryons $\alpha_B = 1.068 (\text{GeV})^2$ [26] and mesons $\alpha_M = 1.092 (\text{GeV})^2$ [33]. Such a behavior is also expected on basis of soft QCD strings in which the strings elongate as they rotate [34]. In the same spirit as in algebraic models of stringlike hadrons [26, 33], we use the mass-squared operator. The spin-flavor part is expressed in a Gürsey-Radicati form [25], i.e. in terms of Casimir invariants of the spin-flavor group chain of Eq. (2)

$$\begin{aligned} M^2 = & M_0^2 + M_{\text{vib}}^2 + \alpha L + a C_{2SU_{\text{sf}}(6)} + b C_{2SU_{\text{f}}(3)} + c C_{2SU_{\text{s}}(2)} \\ & + d C_{1U_{\text{Y}}(1)} + e C_{1U_{\text{Y}}(1)}^2 + f C_{2SU_{\text{f}}(2)} . \end{aligned} \quad (23)$$

The coefficients α , a , b , c , d , e and f are taken from a previous study of the nonstrange and strange baryon resonances [26]. The constant M_0^2 is determined by identifying the ground state exotic pentaquark with the recently observed $\Theta^+(1540)$ resonance. Since the lowest orbital states with $L^P = 0^+$ and 1^- are interpreted as rotational states, there is no contribution from the vibrational term M_{vib}^2 to the mass of the corresponding pentaquark states.

Just as for the harmonic oscillator case, the lowest pentaquark state is a flavor antidecuplet state with spin $s = 1/2$ and isospin $I = 0$, in agreement with the available experimental information which indicates that the $\Theta^+(1540)$ is an isosinglet [3, 4, 6]. In the present calculation, the ground state pentaquark has angular momentum and parity $J^P = 1/2^-$, in agreement with recent work on QCD sum rules [15], lattice QCD [16] and a quark model calculation [17]. It belongs to the $[42111]_{F_2}$ spin-flavor multiplet and the orbital excitation $0_{A_1}^+$. The first excited state at 1599 MeV is an isospin triplet with strangeness $S = +1$ of the 27-plet with the same value of angular momentum and parity $J^P = 1/2^-$. The lowest pentaquark state with positive parity occurs at 1668 MeV. In the absence of a spin-orbit coupling, in this case we have a doublet with angular momentum and parity $J^P = 1/2^+$, $3/2^+$.

The wave function of the ground state pentaquark has the same general structure as that of Eq. (17) for the harmonic oscillator quark model

$$\psi_{A_2} = \left[\psi_{F_1}^c \times [\psi_{A_1}^o \times \psi_{F_2}^{\text{sf}}]_{F_2} \right]_{A_2} . \quad (24)$$

They differ with respect to the treatment of the orbital part of the wave function, but the color-spin-flavor part is the same. Since the orbital wave function has $L_t^P = 0_{A_1}^+$, the magnetic moments only depend on

the spin part, and hence the magnetic moments in a stringlike algebraic model are the same as those for the harmonic oscillator quark model of Eq. (19).

6 Summary and conclusions

The recent experimental evidence for the existence of exotic baryons has prompted an enormous amount of theoretical activity to interpret the data and to help understand its properties. The different approaches can be divided roughly into chiral soliton models which motivated the experiments, QCD related studies (large N_c limit, lattice calculations, sum rules), correlated quark models in which the exotic baryons arise as a quark cluster state, and uncorrelated or constituent quark models in which the exotic pentaquarks are interpreted as $q^4\bar{q}$ bound states.

In this article, we have analyzed the mass spectrum and the magnetic moments of exotic pentaquarks in a constituent quark model. First, we constructed a complete classification scheme based on two conditions: the pentaquark wave function is a color singlet, and is antisymmetric under any permutation of the four quarks. Exotic pentaquark states occur only in the flavor antidecuplet, the 27-plet and the 35-plet. The basis states in correlated quark models form a subset of the ones we have constructed. The precise ordering of pentaquark states in the mass spectrum depends on the choice of a specific dynamical model (Skyrme, chiral potential, Goldstone Boson Exchange, instanton, hypercentral, stringlike, ...). A schematic calculation in a harmonic oscillator quark model shows that the ground state configuration depends on the relative size of the orbital excitations and the spin-flavor splittings.

For the calculation of the photo- and electroproduction cross sections of pentaquarks an important ingredient is the magnetic moment. Hereto we have derived the magnetic moments of the antidecuplet pentaquarks for both parities. For negative parity, they are typically an order of magnitude smaller than the proton magnetic moment, whereas for positive parity there is an additional reduction due to a cancellation between orbital and spin contributions. The magnetic moment of the Θ^+ pentaquark is found to be $0.382 \mu_N$ for negative parity, and $0.089 \mu_N$ for positive parity.

Finally, we investigated the spectroscopy of exotic pentaquarks in an algebraic stringlike quark model which uses a generalized Gürsey-Radicati form for the spin-flavor contributions. The interaction strenghts were taken from previous work on q^3 baryons. As a result, we obtained a ground state antidecuplet pentaquark with $J^P = 1/2^-$ and isospin $I = 0$, in agreement with experimental evidence that the $\Theta^+(1540)$ is an isosinglet. The first excited state at 1599 MeV is a Θ_1 isospin triplet with strangeness $S = +1$ of the 27-plet with the same value of angular momentum and parity $J^P = 1/2^-$. The antidecuplet state with strangeness $S = -2$ and isospin $I = 3/2$ is calculated at an energy of 1956 MeV, to be compared with the recently observed $\Xi_{3/2}^-$ resonance at 1862 MeV [9]. The lowest pentaquark state with positive parity occurs at 1668 MeV. In the absence of a spin-orbit coupling, in this case we have a doublet with angular momentum and parity $J^P = 1/2^+, 3/2^+$. The magnetic moment of the Θ^+ ground state pentaquark is $0.382 \mu_N$, the same value as for the harmonic oscillator quark model.

In conclusion, the spectroscopy of exotic baryons will be a key testing ground for models of baryons and their structure. Especially the measurement of the angular momentum and parity of the $\Theta^+(1540)$ may help to distinguish between different models and to gain more insight into the relevant degrees of freedom and the underlying dynamics that determines the properties of exotic baryons.

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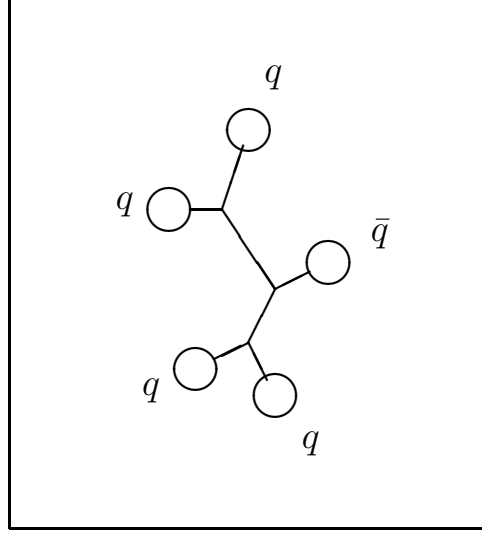


Figure 1: Pentaquark configuration $q^4 \bar{q}$

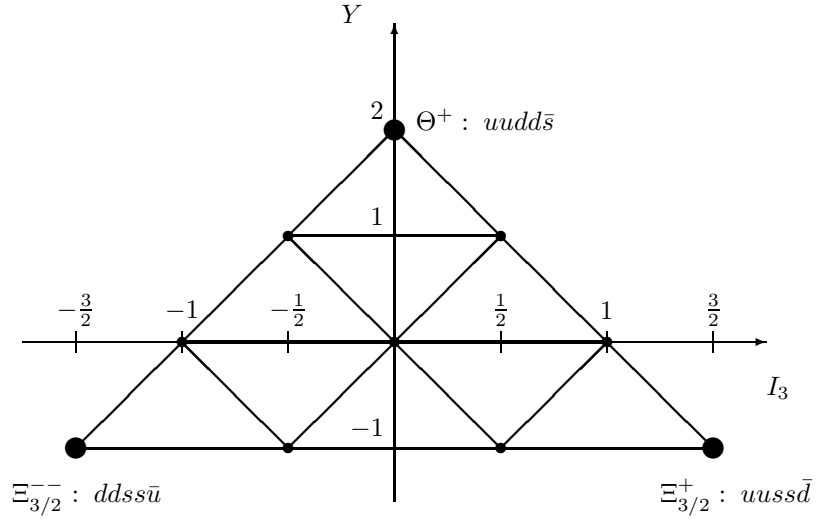


Figure 2: $SU(3)$ flavor multiplet $[33]$ with E symmetry. The isospin-hypercharge multiplets are $(I, Y) = (0, 2), (\frac{1}{2}, 1), (1, 0)$ and $(\frac{3}{2}, -1)$. Exotic states are indicated with \bullet .

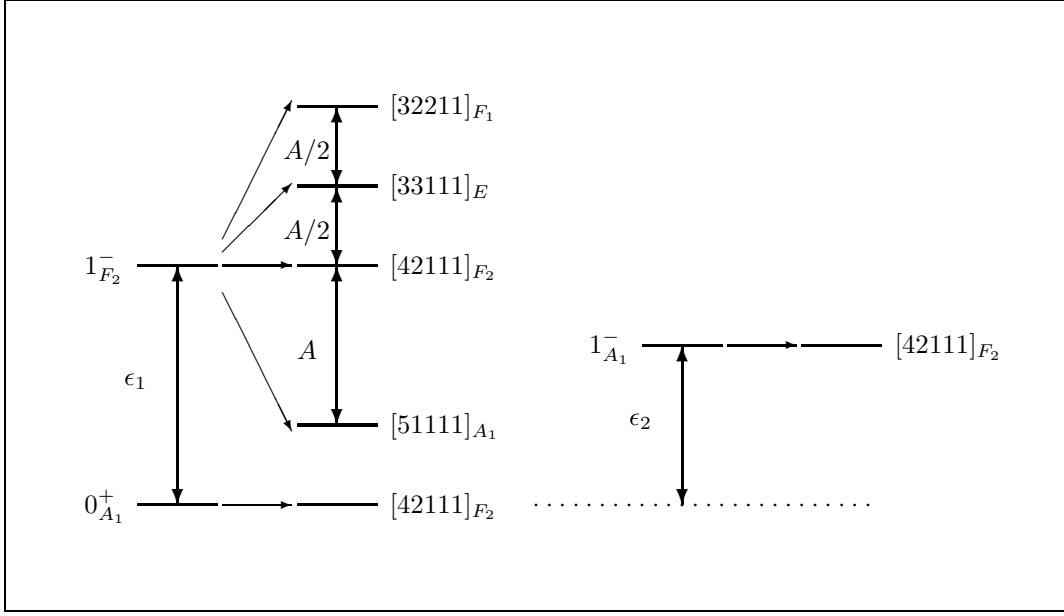


Figure 3: Schematic spectrum of exotic pentaquarks for orbital excitations up to $N = 1$ quantum calculated using Eq. (13) with $A < \epsilon_1$ and $B = C = D = E = 0$. The orbital excitations are labeled by angular momentum and parity L_t^p and the spin-flavor multiplets by $[f]_t$, where the index t gives the permutational symmetry of the four-quark system. The ground state is associated with the $L_t^p = 0_{A_1}^+$ orbital state. The $1_{F_2}^-$ state arises from the relative motion of the four quarks, whereas the $1_{A_1}^-$ state comes from the relative motion between the antiquark and the four-quark system.

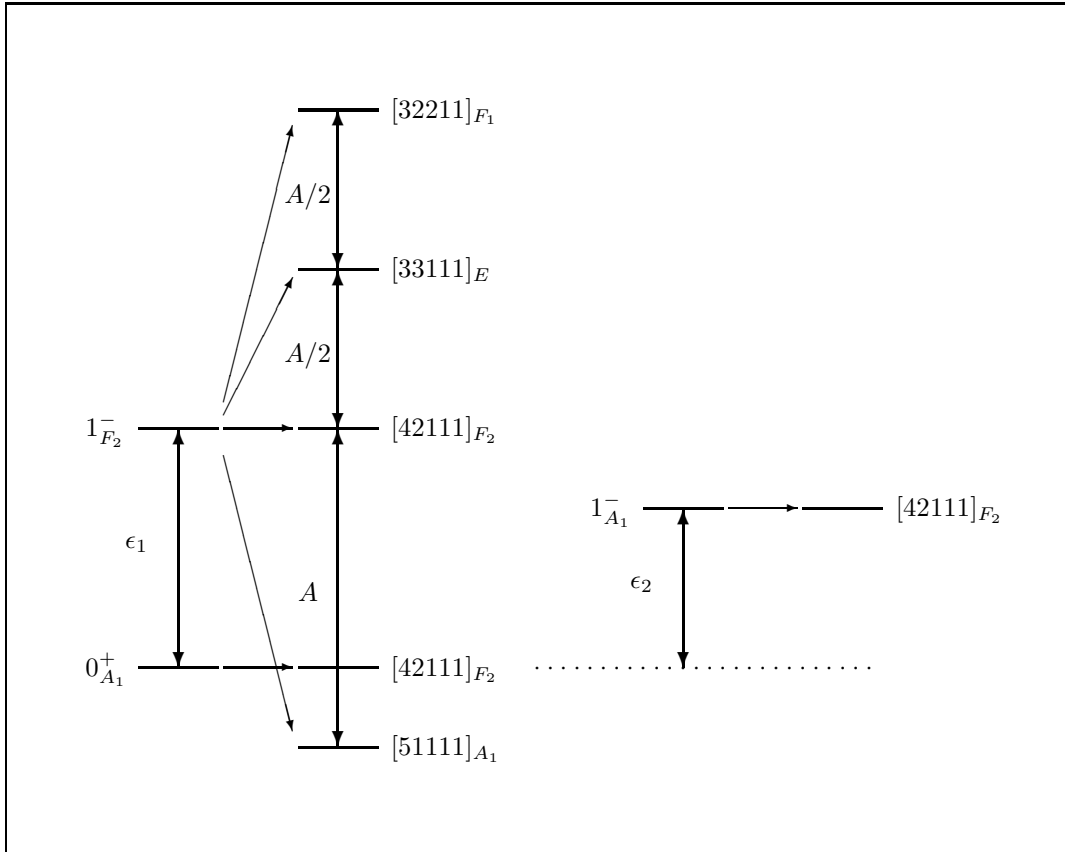


Figure 4: As Figure 3, but for $A > \epsilon_1$. The ground state is associated with the $L_t^p = 1_{F_2}^-$ state.

Table 1: Allowed spin, flavor and spin-flavor pentaquark states

	$qqqq\bar{q}$	Dimension	$S_4 \sim \mathcal{T}_d$
spin	[5]	6	A_1
	[41]	4	A_1, F_2
	[32]	2	F_2, E
flavor	[51]	35-plet	A_1
	[42]	27-plet	F_2
	[33]	antidecuplet	E
	[411]	decuplet	A_1, F_2
	[321]	octet	F_2, E, F_1
	[222]	singlet	F_1
spin-flavor	[51111]	700	A_1
	[41111]	56	A_1, F_2
	[42111]	1134	F_2
	[32111]	70	F_2, E, F_1
	[33111]	560	E
	[32211]	540	F_1
	[22211]	20	F_1, A_2
	[22221]	70	A_2

Table 2: Discrete symmetry of exotic pentaquark states

ψ	ψ^c	ψ^{osf}	ψ^o	ψ^{sf}	Exotic spin-flavor configuration
A_2	F_1	F_2	A_1	F_2	[42111]
A_2	F_1	F_2	F_2	A_1	[51111]
				F_2	[42111]
				E	[33111]
				F_1	[32211]

Table 3: Comparison of magnetic moments in μ_N of exotic antidecuplet pentaquarks with angular momentum $J = \frac{1}{2}$ for both positive and negative parity.

	Positive parity			Negative parity		
	Θ^+	$\Xi_{3/2}^+$	$\Xi_{3/2}^{--}$	Θ^+	$\Xi_{3/2}^+$	$\Xi_{3/2}^{--}$
Present	0.09	0.09	0.09	0.38	0.50	-0.44
[30]	0.12	0.12	-0.24			
	0.20	0.20	-0.40			
	0.30	0.30	-0.60			
[19]	0.18			0.49		
	0.36			0.31		
[31]*				0.12 ± 0.06		
[28, 20, 11]	0.08	-0.06	0.12			
[28, 12]	0.23	0.33	-0.17			
[28, 13]	0.19	0.13	-0.43			
[28, 20, 32]				0.63	0.87	-1.74

* Absolute value